

537.86  
N 76

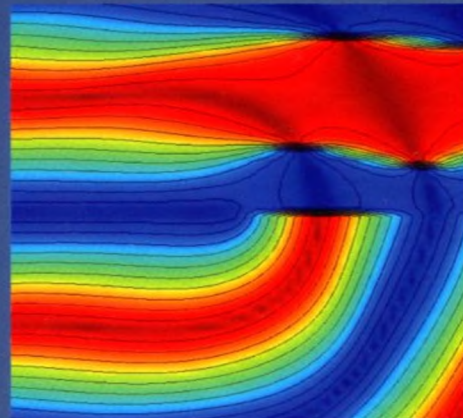
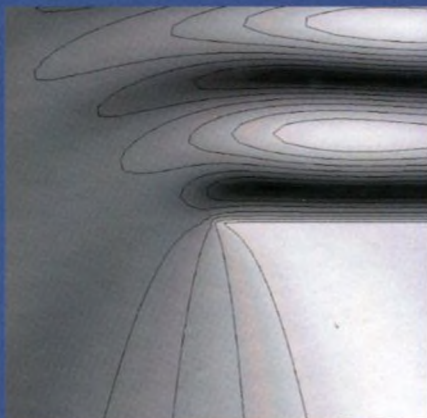


Edited by  
**Eldar I. Veliyev**



# Non-integer Derivative Method in Electromagnetic Wave Scattering Problems

Theory and Applications



**ELDAR I. VELIYEV, ERTUĞRUL KARAÇUHA,  
VASIL TABATADZE, OSMAN DUR**

**NON-INTEGER DERIVATIVE METHOD  
IN ELECTROMAGNETIC WAVE  
SCATTERING PROBLEMS**

**THEORY AND APPLICATIONS**

**EDITED BY ELDAR I. VELIYEV**

**Kharkov  
«Kontrast»  
2019**

UDK 537.86  
N76

*Authors:* Eldar I. Veliyev, Ertuğrul Karaçuha, Vasil Tabatadze, Osman Dur

*Edited by:* Eldar I. Veliyev

Eldar I. Veliyev, Ertuğrul Karaçuha, Vasil Tabatadze, Osman Dur  
N76 **Non-integer** derivative method in electromagnetic wave scattering  
problems. Theory and applications. – Kharkov: «Kontrast», –  
2019-216 p.  
ISBN 978-617-7405-34-3

The book is devoted to the application of the mathematical apparatus of non-integer derivatives to the solution of the diffraction problems by the planar screens.

**UDK 537.86**

ISBN 978-617-7405-34-3

© Eldar I. Veliyev, 2019

---

## CONTENTS

---

Introduction.....	8
<b>Chapter I Fractional solution of the Helmholtz equation.....</b>	<b>25</b>
<i>By Eldar Veliyev</i>	
1.1 Fractional derivative and integral - the main definitions.....	25
1.2 Fractional Green's function.....	27
1.3 Generalization of the Green's theorem using fractional derivatives.....	32
1.4. Fractional boundary conditions.....	34
1.5. Fractional boundary conditions in the problems of reflection.....	35
1.6. Representation of the scattered field.....	41
1.7. Application of the fractional derivatives in the radiation problems.....	42
<b>Chapter II Diffraction problem by the strip with the fractional boundary conditions.....</b>	<b>47</b>
<i>By Eldar Veliyev</i>	
2.1. The method of solution for the fractional integro-differential equation .....	48
2.2. The wave diffraction problem by the strip with the fractional boundary conditions. The E-polarization case.....	54
2.3. The simulation of the boundary conditions of the third kind using the fractional boundary conditions.....	69
2.4. The diffraction problem by the strip with the fractional boundary conditions. The H-polarization case.....	79
2.5 Diffraction problem of the wave by the semi plane with the fractional boundary conditions.....	83
<b>Chapter III Fractional rotor operator.....</b>	<b>91</b>
<i>By Eldar Veliyev</i>	
3.1 Fractalized operators.....	91
3.2 Fractalization of the duality principle of the Maxwell's equations.....	92
3.3 The method for obtaining the fractional operator from the linear operator.....	93
3.4 Fractional rotor - derivation of the representation .....	93
3.4.1 The fractional rotor of the function with one variable.....	96
3.4.2 fractional rotor for the exponential function of the two variables .....	98
3.4.3 The fractional rotor for the exponential function with the three variables .....	99
3.5 The problems of radiation .....	100
3.5.1 The plane wave propagation.....	101

---

3.5.2 Radiation of the current plate .....	102
3.5.3 The polarization of the fractional field.....	104
3.5.4 Radiation by the wire current.....	110
3.5.5 Dipole radiation.....	111
3.5.6 The fractional currents .....	113
3.6. Building a fractional solution in reflection problems .....	114
3.6.1. Anisotropic Impedance Boundary Conditions.....	116
3.6.2. Biisotropic layer.....	124
3.7. Passing through the chiral layer.....	127
<b>Chapter 4 Diffraction by the strip with fractional boundary condition.....</b>	<b>136</b>
<i>by Eldar Veliyev, Ertuğrul Karaçuha, Vasil Tabatadze and Kamil Karaçuha</i>	
Chapter 4.1 The Use of the Fractional Derivatives Approach to Solve the Problem of Diffraction of a Cylindrical Wave on an Impedance Strip.....	136
4.1.1 Formulation of the problem.....	136
4.1.2 Solution of the problem .....	138
4.1.3 Physical characteristics of the scattered field.....	139
4.1.4 Numerical results.....	140
Chapter 4.2 Analysis of Current Distributions and Radar Cross Sections of Line Source Scattering from Impedance Strip by Fractional Derivative Method .....	144
4.2.1 Introduction.....	145
4.2.2 Formulation of the Problem.....	146
4.2.3 Numerical Analysis.....	149
4.2.4 Numerical Results.....	150
4.2.5 Conclusion .....	155
Chapter 4.3 Scattering of a Cylindrical Wave from an Impedance Strip by Using the Method of Fractional Derivatives.....	157
4.3.1 Introduction.....	157
4.3.2 Formulation of The Problem.....	158
4.3.3 Numerical Results.....	161
4.3.4 Conclusion.....	164
Chapter 4.4 Plane wave diffraction by the strip with a complex-ordered fractional boundary condition.....	166
4.4.1 Introduction.....	166
4.4.2 Formulation of the Problem.....	167
4.4.3 Investigation of Convergence in Numerical Analysis.....	170

---

4.4.4 Physical Characteristics of the Electric Field .....	171
4.4.5 Numerical Analysis and Results .....	172
4.4.6 Conclusion.....	180
<b>Chapter 5 Diffraction by two strips with fractional boundary condition.....</b>	<b>183</b>
<i>by Eldar Veliyev, Ertuğrul Karaçuha, Vasil Tabatadze and Kamil Karaçuha</i>	
Chapter 5.1 The Fractional Derivative Approach for diffraction problems: Plane Wave Diffraction by Two Strips with Fractional Boundary Conditions.....	183
5.1.1 Introduction.....	183
5.1.2 Theoretical Part.....	184
5.1.3 Results of Numerical Simulations .....	189
5.1.4 Conclusion.....	201
Chapter 5.2 The plane wave diffraction problem solution by the two strips with different fractional boundary conditions.....	203
5.2.1 Introduction.....	203
5.2.2 Formulation of the Problem.....	204
5.2.3 Results of the Numerical Simulation.....	207
5.2.4 Conclusion.....	214