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**MATHEMATICAL  
THEORY  
OF HIGHER-  
ORDER  
DEGENERATE  
EVOLUTION  
MODELS**

NATIONAL ACADEMY OF SCIENCES OF UKRAINE  
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«UKRAINIAN SCIENTIFIC BOOK  
INA FOREIGN LANGUAGE»*

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This book is devoted to the study of solvability and qualitative behaviour of generalised solutions to initial boundary value problems for high-order nonlinear parabolic equations and systems. The eventual goal of this book is to present with all rigorous details the new applications of energy-entropy methods starting from one-dimensional problems discussed in Chapters 1-3, progressing to an advanced level in considering these methods for thin-film type systems in Chapter 4, 5 and finally showing all the steps of the qualitative analysis for problems in multi-dimensional domains in Chapter 6. While we expect our readers to be familiar with a parabolic PDE theory and also to have some knowledge of classical functional analysis we do not assume any background in non-linear PDE analysis and provide enough details to learn the methods from the book.

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