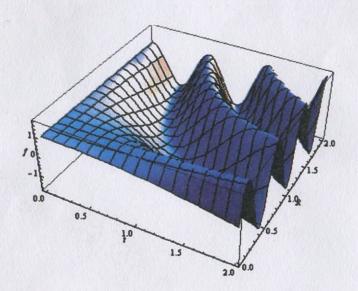
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# V.Z. GRISTCHAK, D.D. GRISTCHAK, Yu.A. FATIEIEVA

# HYBRID ASYMPTOTIC METHODS THEORY AND APPLICATIONS



## V. Z. GRISTCHAK, D. D. GRISTCHAK, Yu. A. FATIEIEVA

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The monograph is devoted to some theoretical improvements in the theory of hybrid asymptotic approach and its applications in the sphere of mathematical modeling for nonlinear dynamics of modern structures. An approximate analytical approach is based on the hybrid perturbation and phase integral (or WKB-Galerkin) asymptotic methods. New solutions of given mechanical objects are given. For the experts in the mathematical physics, deformable media mechanics and adjacent areas, to the lecturers and post-graduate students of the appropriate specialties will be useful as well.

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