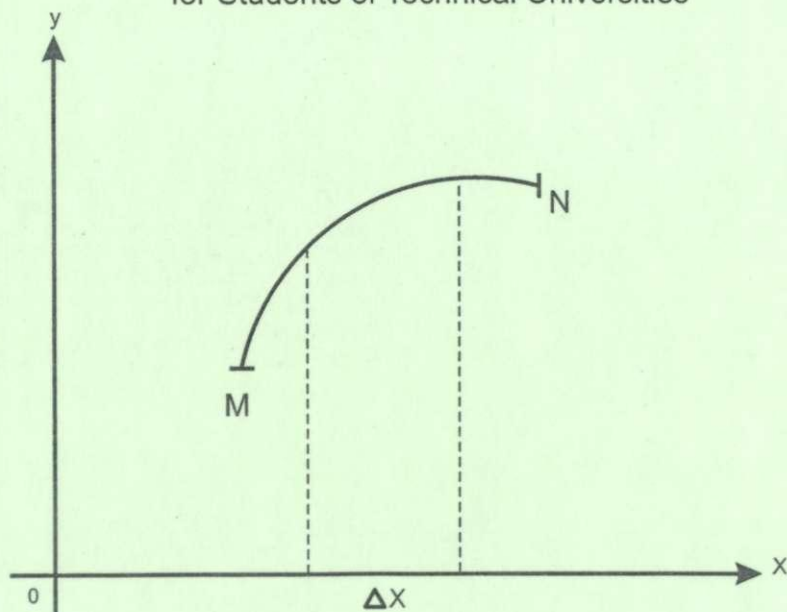


L. V. Kurpa
T. V. Shmatko

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DIFFERENTIAL AND INTEGRAL CALCULUS FOR FUNCTIONS WITH SEVERAL VARIABLES

The Educational Textbook
for Students of Technical Universities



МІНІСТЕРСТВО ОСВІТИ І НАУКИ,
МОЛОДІ ТА СПОРТУ УКРАЇНИ
НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ
«Харківський політехнічний інститут»

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ДИФЕРЕНЦІАЛЬНЕ ТА ІНТЕГРАЛЬНЕ ЧИСЛЕННЯ ФУНКЦІЙ БАГАТЬОХ ЗМІННИХ

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для студентів технічних спеціальностей

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Призначено для студентів та викладачів технічних спеціальностей.

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Theoretical material of differential calculus of several variables, multiple and curvilinear

integrals is proposed This educational textbook includes problems that are recommended to solve as classwork and as homework. Variants of individual tests are created for each subject.

For students and teachers of technical specialties.

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