



**G.P. Butsan**

# Introduction to Probability Theory



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Вступ  
до теорії  
ймовірностей

*ПРОЕКТ*  
*"УКРАЇНСЬКА НАУКОВА КНИГА*  
*ІНОЗЕМНОЮ МОВОЮ"*

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This book is intended even if partially to compensate the absence of some short introduction to Probability Theory in English in Ukraine and can be useful to both students of Ukrainian technical universities and their teachers, as well as to all technicians and engineers whose work concerns the theory. This book deals with the typical problems which occur in practice and contemporary technologies as well as gives the initial acquaintance with Probability Theory which requires consideration of great deal of examples that would help develop a special probability-theoretical and probabilistic intuition, and the ability to connect abstract ideas with practical situations. The presentation of elements of the theory of random processes and elements of the statistics was mainly determined by the need of students of technical universities and their level of knowledge of mathematical analysis and elements of functional analysis in the frame of training programs for such universities and, respectively, by the number of training hours for the presentation of the subject and is accessible to undergraduates.

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# CONTENT

PREFACE . . . . .	5
INTRODUCTION . . . . .	7
CHAPTER 1	
<b>THE MAIN NOTIONS OF THE THEORY</b>	
§ 1.1. The notion of event . . . . .	11
§ 1.2. The intuitive (or classical) notion of probability. . . . .	15
§ 1.3. Geometrical approach to the notion of probability. . . . .	19
§ 1.4. Conditional probability. . . . .	22
§ 1.5. Some useful practical formulas . . . . .	24
§ 1.6. The notion of independence . . . . .	29
§ 1.7. Repeating experiments . . . . .	31
§ 1.8. Axiomatic approach . . . . .	39
<i>Exercises.</i> . . . .	42
CHAPTER 2	
<b>RANDOM VARIABLES</b>	
§ 2.1. Random variables and their $\sigma$ -algebras . . . . .	46
§ 2.2. Distribution function . . . . .	47
§ 2.3. Notion of Lebesgue integral . . . . .	49
§ 2.4. The main numerical characteristics for random variables . . . . .	58
§ 2.5. The distribution density for a random variable. . . . .	66
§ 2.6. Some other parameters of random variables which sometimes are used in practice. . . . .	74
<i>Exercises.</i> . . . .	76
CHAPTER 3	
<b>THE SYSTEM OF THE RANDOM VARIABLES</b>	
§ 3.1. Concept of the system of random variables_____	79
§ 3.2. Distribution law of the system of random variables. Distribution table . . . . .	80
§ 3.3. Distribution function for the system of two random variables . . . . .	81
§ 3.4. The distribution density for the system of two random variables . . . . .	83
§ 3.5. Distribution density of individual values of the system. Conditional distribution laws. . . . .	87
§ 3.6. Dependent and independent random variables . . . . .	90

§ 3.7. Numerical characteristics for the system of two random variables. Correlation moment correlation coefficient . . . . . 93

§ 3.8. Function and density for the distribution of a system with arbitrary number of random variables. . . . . 96

§ 3.9. Numerical characteristics for the system of arbitrary finite number of random variables. . . . . 98

§ 3.10. Normal distribution on the plane . . . . . 100

*Exercises.* . . . . 106

CHAPTER 4

**FUNCTIONS FROM RANDOM VARIABLES**

§ 4.1. The distribution law for one function from the system of random variables 109

§ 4.2. Distribution of vector transformation of the system of two random variables 113

§ 4.3. Determining the mathematical expectation for the function from random variable. . . . . 119

§ 4.4. Determination of dispersion for random variables as function from random vectors. . . . . 124

§ 4.5. Next important properties of correlation moment and correlation coefficient . . . . . 128

§ 4.6. Complex random variable. . . . . 131

§ 4.7. Characteristic functions . . . . . 133

*Exercises.* . . . . 137

CHAPTER 5

**LIMIT THEOREMS**

§ 5.1. Notion of convergences for random variables and Chebyshev inequality .. 140

§ 5.2. The laws of large numbers . . . . . 142

§ 5.3. Central limit theorem . . . . . 147

*Exercises.* . . . . 150

CHAPTER 6

**MARKOV CHAINS**

§ 6.1. General idea . . . . . 152

§ 6.2. Transition matrix . . . . . 153

§ 6.2. Theorem for limit probabilities . . . . . 154

*Exercises.* . . . . 156

CHAPTER 7

**RANDOM PROCESSES**

§ 7.1. Definition of random process . . . . . 157

§ 7.2. Multidimensional densities of probability . . . . . 158

§ 7.3. Mathematical expectation and dispersion of random process . . . . .	160
§ 7.4. Correlation function of random process . . . . .	161
§ 7.5. Moments of higher orders . . . . .	165
§ 7.6. Examples of random processes . . . . .	165
§ 7.7. Complex random processes . . . . .	168
§ 7.8. Operations performed on random processes . . . . .	169
§ 7.9. Canonical expansion of random process . . . . .	175
§ 7.10. Stationary random processes . . . . .	178
§ 7.11. Ergodic property of the stationary random process . . . . .	180
§ 7.12. Spectral decomposition of stationary random process on the finite interval . . . . .	183
§ 7.13. Spectral decomposition of stationary random process on infinite interval. Spectral density of stationary random process . . . . .	186
§ 7.14. Examples of stationary random processes . . . . .	190
§ 7.15. Transformation of stationary random process by linear system . . . . .	193
<i>Exercises.</i> . . . . .	197

CHAPTER 8

**ELEMENTS OF MATHEMATICAL STATISTICS**

§ 8.1. Subject matter of mathematical statistics . . . . .	202
§ 8.2. Parent population and sample . . . . .	203
§ 8.3. Statistical array. Statistical distribution function . . . . .	204
§ 8.4. Statistical population. Histogram . . . . .	205
§ 8.5. Numerical characteristics of statistical distribution . . . . .	207
§ 8.6. Properties of point estimates . . . . .	208
§ 8.7. Determination of approximate value of measured quantity and approximate variance value in case of direct uniformly precise measurements . . . . .	209
§ 8.8. Determination of approximate value of the measured quantity in case of unequal measurements . . . . .	213
§ 8.9. Confidence interval. Confidence probability . . . . .	216
§ 8.10. Construction of confidence interval for mathematical expectation of random variable distributed by normal law. Student's distribution . . . . .	218
§ 8.11. Determination of approximate values of numerical characteristics for the system of two variables . . . . .	221
§ 8.12. Maximum likelihood method for finding the estimates of distribution parameters . . . . .	223
§ 8.13. Smoothing the experimental dependences . . . . .	225
§ 8.14. The least squares method . . . . .	226
§ 8.15. Statistical check of hypotheses . . . . .	229
§ 8.16. Goodness of fit test . . . . .	236
<i>Exercises.</i> . . . . .	238
APPLICATIONS . . . . .	241
REFERENCES . . . . .	245