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VOLODYMYR
SHCHEDRYK

ARITHMETIC OF MATRICES OVER RINGS

$PAAQA = \text{diag}(a_1, a_2, \dots, a_n) = E$

$G_\Phi = \{H \in GL_n(R) \mid \exists K \in GL_n(R) : H\Phi = \Phi K\}$

h_{11}	h_{12}	\dots	$h_{1,n-1}$	h_{1n}
$\frac{\varphi_2}{\varphi_1} h_{21}$	h_{22}	\dots	$h_{2,n-1}$	h_{2n}
$\frac{\varphi_n}{\varphi_1} h_{n1}$	$\frac{\varphi_n}{\varphi_2} h_{n2}$	\dots	$\frac{\varphi_n}{\varphi_{n-1}} h_{n,n-1}$	h_{nn}

$PAAQA = \text{diag}(a_1, a_2, \dots, a_n)$

NATIONAL ACADEMY
OF SCIENCES OF UKRAINE
PIDSTRYHACH INSTITUTE FOR APPLIED
PROBLEMS OF MECHANICS
AND MATHEMATICS
OF THE NAS OF UKRAINE

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SHCHEDRYK

ARITHMETIC OF MATRICES OVER RINGS

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«UKRAINIAN SCIENTIFIC BOOK»
IN A FOREIGN LANGUAGE

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The book is devoted to investigation of arithmetic of the matrix rings
over certain classes of commutative finitely generated principal ideals do-
mains. We mainly concentrate on constructing of the matrix factorization
theory. We reveal a close relationship between the matrix factorization and
specific properties of subgroups of the complete linear group and the special
normal form of matrices with respect to unilateral equivalence. The properties
of matrices over rings of stable range 1.5 are thoroughly studied.

The book is intended for experts in the ring theory and linear algebra,
senior and post-graduate students.

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