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DIFFERENTIAL AND INTEGRAL CALCULUS FOR ONE VARIABLE FUNCTIONS

The Educational Textbook
for Students of Technical Universities

ДИФЕРЕНЦІАЛЬНЕ ТА ІНТЕГРАЛЬНЕ ЧИСЛЕННЯ ФУНКЦІЙ ОДНІЄЇ ЗМІННОЇ

Навчальний посібник
для студентів технічних спеціальностей

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В навчальному посібнику розглядається теоретичний матеріал з таких розділів вищої математики: теорія границь, неперервність функцій, диференціальне та інтегральне числення для функцій однієї змінної. Посібник містить в собі доведення всіх необхідних теорем, передбачених робочою програмою з математичного аналізу стосовно розглянутих тем, та ілюстрацію їх на конкретних прикладах.

Призначено для студентів технічних університетів, які вивчають вищу математику англійською мовою.

Курпа Л. В.

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The educational textbook includes theoretical material of the following sections of the higher mathematics: Limits theory, continuity of functions, differential and integral calculus for functions of one variable. The manual contains all necessary proof of theorems provided the work program of mathematical analysis regarding topics discussed, and their illustration on concrete examples.

The educational textbook would be useful for students of technical universities, who are studying the higher mathematics in English.

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